The Lowly Worm Climbs Up a Winding Stair: Teaching the Roles of Randomness in Scientific Inference

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Cognitive Challenges of Scientific Inference

Scientists must incorporate both the **reliability of evidence** and the **scope of inference**, yet both are **complex phenomena** that can be difficult to understand.

Difficulties with reliability of evidence

- Tversky & Kahneman (1971) found quantitative psychologists **vastly overestimated replicability** of findings from small studies
- Underpowered studies lead to many **"voodoo correlations"** (Fiedler, 2011) being reported in the literature
- Adults **frequently neglect sample size** (see Lem et al., 2011), which statistical training reduces but does not eliminate (e.g., Fong, Krantz, & Nisbett, 1986)

Difficulties with scope of inference

- Introductory statistics students struggle to understand when **study design** warrants generalizing to a population or concluding causation (e.g., Derry et al., 2000)
- News media reporting on research routinely **overgeneralizes** and adds **inappropriate causal attributions** (Schield, 2010)
- Scientists face many **cognitive biases** (Nuzzo, 2015) and **overstate their own findings** (see Utts, 2003)

Statistics education research can contribute to metascience's investigation of how scientists **interpret and treat evidence**:

- Students face **unique challenges** in learning statistics that require specialized pedagogical research (Garfield & Ben-Zvi, 2007)
- Statistics education research provides perspectives on **students' understanding at all levels**, from kindergarten to the workforce
- Our studies of introductory college statistics' students understanding can illuminate the challenges in **conceptualizing and adequately addressing threats to internal and external validity**.

Teaching Inference with Simulation

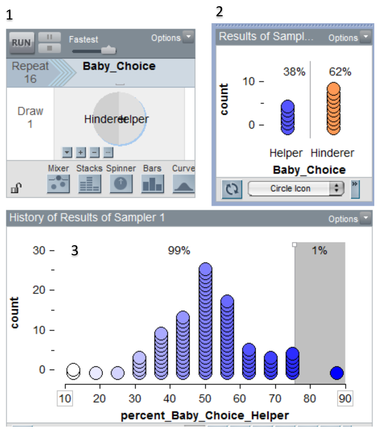
Statistics educators increasingly use **simulation and resampling techniques** to teach frequentist inference (Tintle et al., 2015):

- Simulation, bootstrap resampling, and rerandomization may help students focus on **the unified logic** of the sampling distribution (Cobb, 2007)
- Displaying uncertainty via concrete realizations of a stochastic process may **map on to human cognitive faculties** more naturally than abstract distributions (Gigerenzer & Hoffrage, 1995; Hullman et al., 2015)
- Simulation-based introductory statistics curricula have **some evidence of providing advantages** in student understanding and outcomes (Tintle et al., 2012)

Example simulation-based activity (from Zieffler et al., 2017)

In Hamline & Bloom (2007), 14 out of the 16 (87.5%) babies chose the "Helper" toy. *Does this study provide evidence that infants notice and prefer the "Helper"?*

1. **Students set up null model visually** in TinkerPlots: e.g., a spinner with 50% helper and 50% hinderer, or could be two balls "helper" and "hinderer" chosen with replacement.
2. **Students plot the result of a single trial** under null hypothesis.
3. **Students collect the percentage of means of many trials**, and evaluate how rare results at or above 87.5% would be under the null model.

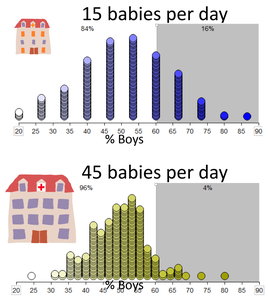


Study 1: Reliability of Evidence (Brown, 2019)

Background

*In one year, which hospital do you expect to have **more days with more than 60% boys**?*

- *The larger hospital*
- *The smaller hospital*
- *About the same* (Kahneman & Tversky, 1972)



As **sample size** increases, we **grow more certain** that a statistic is close to the population value. This fundamental principle is called the **Empirical Law of Large Numbers** (Freudenthal, 1972)

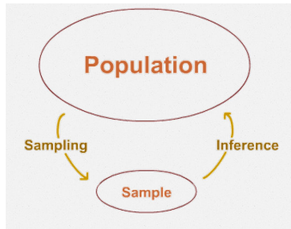
Common confusions include:

- Ignoring sample size, believing that similarity to population is only factor
- Confusing frequency and sampling distributions
- Attending more to the ratio of the sample to the population size

Prior interventions:

- Demonstrate that sample size is responsive to training
- Rely on learning a rule (Fong, Krantz, & Nisbett, 1986) and/or inferring a principle from demonstrations (Chance, delMas & Garfield, 2004; Sedlmeier, 1999)
- Do not support students in exploring the *mechanisms* of power and precision

Study 2: Scope of Inference (Fry, 2017)



According to statistics education recommendations (e.g., Utts, 2003), students should understand:

- **Random sampling** tends to produce representative samples, allowing for **generalization** to a population.
- **Random assignment** tends to balance out confounding variables between groups, helping to enable **cause-and-effect** conclusions.

Students have difficulties such as:

- Confusion between random sampling and random assignment (Derry et al., 2000)
- Disbelief that random assignment can help enable causal claims (Sawilowsky, 2004)
- Believing that larger samples are always better than smaller samples, regardless of method (Wagler & Wagler, 2013)

Methods

***Growing Certain*: a series of activities to provide support for understanding mechanisms of power and precision**

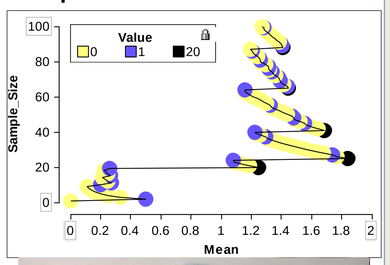
Primary conceptual targets:

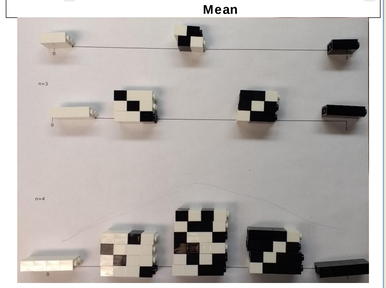
Swamping:

- means of large samples are less influenced by extreme values than small samples (Well et al., 1990)
- Students explore through randomly drawing additional values and watching decreasing impact of a value on mean (right)

Heaping

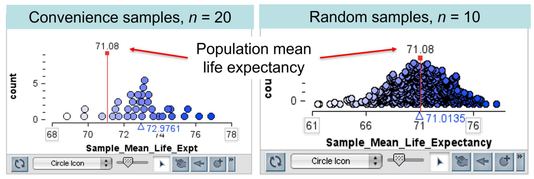
- increasing % of ways to get a sample mean near true mean (inspired by Abrahamson, 2006)
- Participants create both empirical and theoretical sampling distributions and examine permutations





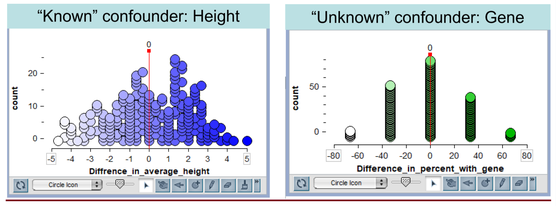
Participants were 5 volunteer introductory statistics students, who participated in 6 videorecorded hours of clinical interviews. One focal participant was coded for mechanistic reasoning (Russ et al. 2008)

Two-and-a-half week *Study Design* unit implemented in undergraduate introductory statistics course



Sampling and bias: Students compare sample means from larger convenience samples (biased method), to sample means from smaller random samples (unbiased method) .

Random assignment and balancing confounding variables: Students contrast purposeful assignment with random assignment, observing how random assignment tends to balance differences in confounding variables

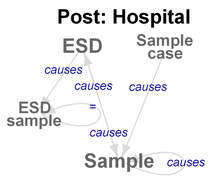


- One activity specifically designed to have students carry out both random sampling and random assignment, and distinguish between purposes of each
- Pretest/posttest taken by n = 125 students, qualitative data analysis of group quizzes and individual homework assignments

Results

Nearly all **students successfully used swamping reasoning**, but application depended on context and student

- Students were able to recognize that having more variability, or a mean closer to hypothesized value, meant a larger sample size was needed
- **Students had difficulty reasoning about many samples and often argued based on results from a single sample**
- Students **successfully drew on their everyday experiences** (getting an F at the end of a course as opposed to beginning) to explain Empirical Law of Large Numbers



Coded mechanistic relationships between entities for the hospital problem for one participant.

ESD = Empirical Sampling Distribution.

Limitations

- Sample size of 1!
- Mechanistic coding had 1 coder
- Attribution to activities unclear
- Low ecological validity
- Not classroom-scalable

"But looking at it **individually** [...] I would say still **the smaller hospital, just because it's more likely to deviate**, while the bigger hospital [...] doesn't go up and down quite as drastically. "

Overall, **evidence of gains** in learning goals related to study design and conclusions, especially those related to understanding:

- The purpose of random assignment
- Correlation does not imply causation

A small, but noticeable portion of students experience difficulties such as:

- **Confusion** between random sampling and random assignment
- **Giving sample size more importance** than sampling method

Limitations

- Neither randomly assigned nor sampled!
- Pretest given just before unit and posttest given just after unit: Did not measure student knowledge at beginning or end of course
- Instrument had evidence of low reliability

Sample item showing some evidence of lingering confusion between random sampling and random assignment:

Researchers conducted a survey of 1,000 randomly selected adults in the United States and found a strong, positive, statistically significant correlation between income and the number of containers the adults reported recycling in a typical week.

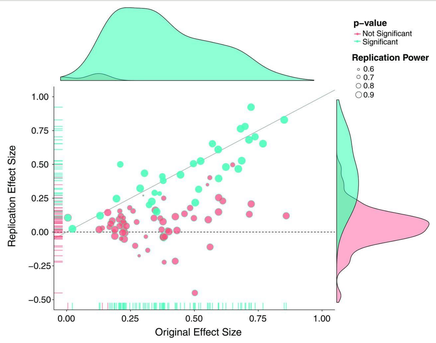
Can the researchers conclude that higher income causes more recycling among U.S. adults? Select the best answer from the following options.

Response	Pretest (%)	Posttest (%)
No, the sample size is too small to allow causation to be inferred.	35.2	7.2
No, the lack of random assignment does not allow causation to be inferred.	**28.0**	**77.6**
Yes, the statistically significant result allows causation to be inferred.	12.0	3.2
Yes, the sample was randomly selected, so causation can be inferred.	**24.8**	**12.0**

Future Directions in Teaching Scientific and Statistical Inference

Statistical Understanding and the Reproducibility Crisis

- **Better instruction** is needed to support scientists and non-scientists in statistical reasoning. Simulation-based representation may be one useful tool.
- **Much more research is needed** into understanding how these crucial types of reasoning can be improved.
- Another crucial topic is people's understanding of **the implications of multiple testing and selective inference** (e.g. Benjamini & Hochberg, 1995), another major contributor to the reproducibility crisis



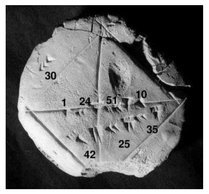
(Open Science Collaboration, 2015)

Topics for Future Research

- What **aspects of a curriculum** are most helpful for students' learning about reliability and scope of inference?
- How well do students **retain conceptual understanding** at the end of the course, or after the course?
- How can **single and multiple samples** be bridged effectively? Would Hypothetical Outcome Plots (Hullman et al., 2015) help?
- What are *researchers*' mechanistic reasoning about reliability and scope of inference?
- How could these concepts be taught in a **shorter unit**, or to researchers outside of a classroom setting?
- Does mechanistic reasoning support decision-making?

Building more experience with statistical processes

- These complex concepts may need more experience beyond standard statistical courses
- **Earlier, deeper, and widely accessible** introductions to statistical reasoning can start in early primary school (Franklin et al., 2005)
- Reliability and scope of inference can be **integrated into science education** to give students more experience
- Arithmetic has moved from an **elite** to a **basic literacy skill**; it may be time for the same transition for reasoning with data and uncertainty



Babylonian clay tablet YBC 7289
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