Sensitivity Analysis for Publication Bias in Meta-Analyses

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Introduction

- Existing methods (funnel plot methods, selection models) focus on **estimating publication bias**.
- We focus instead on **conducting sensitivity analyses**, enabling easy-to-report summary measures that help calibrate confidence in a meta-analysis.
- Also enables better performance in small meta-analyses, with non-normal true effects, or with clustering.
- We ask: **How severe would publication bias have to be in order to "explain away" results of a meta-analysis?**

Assumed model of publication bias

Affirmative studies (i.e., $\widehat{\theta} > 0$ and $p < 0.05$) are η-fold more likely to be published than **non-affirmative** studies (i.e., $\widehat{\theta} \leq 0$ or $p \geq 0.05$). Assume publication independent of $\widehat{\theta}$ within affirmative and within non-affirmative studies.

Fixed-effects sensitivity analysis

$\eta\left(\widehat{\mu}, q\right)$: The value of η needed to attenuate the meta-analytic point estimate ($\widehat{\mu}$) to a smaller value, q. $\eta\left(\widehat{\mu}^{lb}, q\right)$: The value of η needed to attenuate the lower 95% CI limit ($\widehat{\mu}^{lb}$) to a smaller value, q.

Bias-corrected model upweights non-affirmative studies by η:

$$\widehat{\mu}_\eta = \left(\underbrace{\sum_{i \in \mathcal{N}} \frac{\eta}{\sigma_i^2} \widehat{\theta}_i}_{\text{non-affirmatives}} + \underbrace{\sum_{j \in \mathcal{A}} \frac{1}{\sigma_j^2} \widehat{\theta}_j}_{\text{affirmatives}} \right) \left(\underbrace{\sum_{i \in \mathcal{N}} \frac{\eta}{\sigma_i^2}}_{\text{non-affirmatives}} + \underbrace{\sum_{j \in \mathcal{A}} \frac{1}{\sigma_j^2}}_{\text{affirmatives}} \right)^{-1}$$

$$\widehat{\text{Var}}\left(\widehat{\mu}_\eta\right) = \frac{\eta^2 \nu_{\mathcal{N}} + \nu_{\mathcal{A}}}{\left(\eta \nu_{\mathcal{N}} + \nu_{\mathcal{A}}\right)^2}$$

Notation

$\widehat{\theta}_i$ = point estimate in i^{th} study; σ_i^2 = SE^2 in i^{th} study; $\mathcal{N}$ = set of non-affirmative studies; $\mathcal{A}$ = set of affirmative studies; for some set $\mathcal{S}$: $\bar{y}_{\mathcal{S}} = \sum_{i \in \mathcal{S}} \frac{1}{\sigma_i^2} \widehat{\theta}_i$ and $\nu_{\mathcal{S}} = \sum_{i \in \mathcal{S}} \frac{1}{\sigma_i^2}$.

Main point

These methods enable statements like:

For publication bias to shift the observed meta-analytic estimate to the null, "significant" positive results would need to be at least 30-fold more likely to be published than negative or "nonsignificant" results.

Large values $\Rightarrow$ **robust** to publication bias.

Preprint: **https://osf.io/s9dp6/**

R package: `PublicationBias`

Solve for η needed to attenuate $\widehat{\mu}$ to q:

Publication bias required to attenuate point estimate to q

$$\eta\left(\widehat{\mu}, q\right) = \frac{\nu_{\mathcal{A}} q - \bar{y}_{\mathcal{A}}}{\bar{y}_{\mathcal{N}} - \nu_{\mathcal{N}} q}$$

e.g., $\eta\left(\widehat{\mu}, 0\right) = -\bar{y}_{\mathcal{A}} / \bar{y}_{\mathcal{N}}$ (attenuate to null)

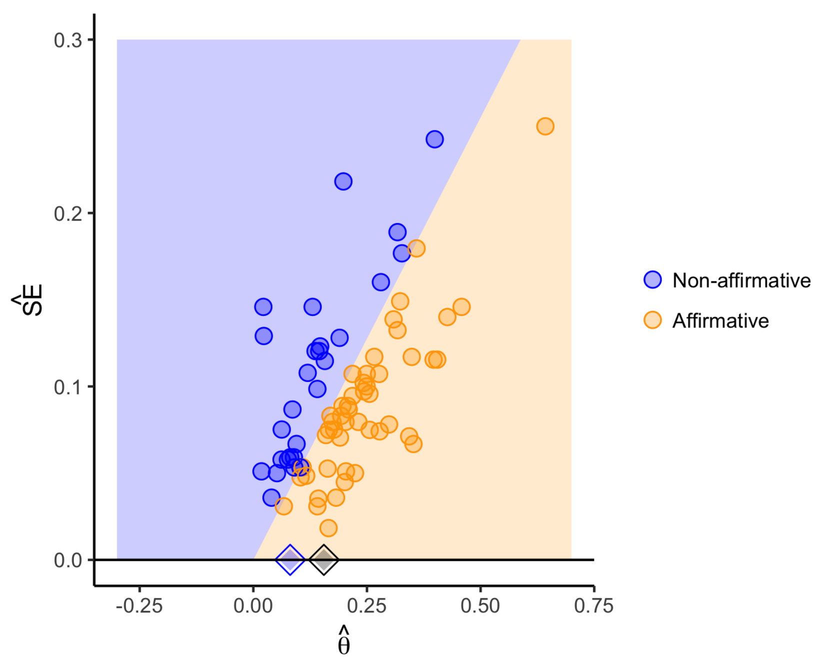
Random-effects sensitivity analysis

We extended GEE-like methods (Hedges, Tipton, Johnson, 2010) to yield sensitivity analyses that accommodate **non-normal true effects, small meta-analyses, and clustering**. Can then obtain $\eta\left(\widehat{\mu}, q\right)$ with a grid search (automated in package).

Worst-case meta-analysis

For a **worst-case point estimate** under maximal publication bias, we can simply **meta-analyze only the non-affirmative studies**. This arises from letting $\eta \to \infty$.

"Significance funnel" plot



Blue diamond: estimate in non-affirmative studies.
Black diamond: estimate in all studies.

Can accompany sensitivity analyses with this modified funnel plot.
Standard funnel can mislead if publication bias operates on p-values instead of effect sizes.

Violent video games and aggression

Meta-analysis of 75 studies (Anderson, Shibuya, Ihori, et al., 2010) found playing violent video games associated with increased aggressive behavior. Debate continues regarding effects of publication bias.

Uncorrected	$\widehat{\mu}$ [95% CI]
Fixed-effects	0.15 [0.14, 0.17]
Robust random-effects	0.18 [0.15, 0.20]
Worst-case	
Fixed-effects	0.08 [0.05, 0.11]
Robust random-effects	0.08 [0.05, 0.12]

Uncorrected and worst-case point estimates (Pearson's r).

Model	$\eta(\widehat{\mu}, 0)$	$\eta(\widehat{\mu}^{lb}, 0)$	$\eta(\widehat{\mu}, q)$	$\eta(\widehat{\mu}^{lb}, q)$
Fixed-effects	N.P.	N.P.	12	5
Robust random-effects	N.P.	N.P.	30	4

Severity of publication bias (η) required to attenuate $\widehat{\mu}$ or $\widehat{\mu}^{lb}$ to null or to $q = 0.10$ on the Pearson's r scale. "N.P." ("not possible") indicates that no value of η could sufficiently attenuate the statistic.